Exam. Code : 103203
Subject Code : 1107

B.A./B.Sc. 3rd Semester (Batch 2020-23)

MATHEMATICS

Paper—I (Analysis)

Time Allowed—3 Hours] [Maximum Marks—50

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Prove that a monotonically decreasing sequence is convergent if and only if it is bounded below. Moreover it converges to g.l.b. of range.

   (b) Prove that sequence defined by $x_1 = 1$, $x_{n+1}\sqrt{6+x_n}$ is convergent and converges to positive root of $y^2 - 9 = 0$.

2. (a) State and prove Cauchy's first theorem on limits.

   (b) Prove that the sequence $\{x_n\}$ where $x_n = \sum_{i=1}^{n} \frac{1}{2i-1}$ is not convergent.

SECTION—B

3. (a) Test the convergence and divergence of the series

$$1+\frac{2}{1}\frac{1}{2}+\frac{2}{1}\frac{4}{3}\frac{1}{3}+\frac{2}{1}\frac{4}{3}\frac{6}{5}\frac{1}{4}\ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Using the integral test, discuss the convergence of the series

$$\sum_{n=2}^{\infty}\frac{n}{n(\log n)^{p}}$$

4. (a) Prove that the series $\sum_{n=1}^{\infty}\left(q^{\frac{1}{n}}-1\right)$ is convergent if and only if $q \neq 1$, $q > 0$.

(b) Show that the series $\sum_{n=1}^{\infty}\left[\frac{1.3.5.....(2n-1)}{2.4.6........(2n)}\right]^{p} x^{n}$ is convergent if $x < 1$, and diverges if $x > 1$ and for $x = 1$, converges for $p > 2$ and diverges for $p \leq 2$.

SECTION—C

5. (a) Prove that every continuous function on [a, b] is Riemann integrable on [a, b].

(b) Define a function f on [0, k] where k is positive integer as follows;

$$f(x)=\begin{cases}0 & \text{when x is an integer}\\ 1 & \text{otherwise}\end{cases}$$

Then f is Riemann integrable on [0, k].

6. (a) If $f(x)=\begin{cases}\cos x & \text{when x is rational}\\ \sin x & \text{otherwise}\end{cases}$ $0 \leq x \leq \pi/4$,

is f Riemann integrable on $[0, \pi/4]$?

(b) Prove that f is Riemann integrable on [a, b] if and only if for given $\in > 0 \exists$ a partition P of [a, b] such that $U(P, f) - L(P, f) < \in$.

SECTION—D

7. (a) State and prove relation between Beta and Gamma functions.

(b) Test the convergence of $\int_{0}^{\pi}\frac{\sqrt{x}}{\sin x}dx$.

8. (a) Evaluate :

$$\int_{0}^{\infty}\frac{x^{2}}{1+x^{4}}dx$$

(b) Show that $\int_{0}^{1}x^{\frac{-1}{3}}(1-x)^{\frac{-2}{3}}(1-2x)^{-1}dx=\frac{1\beta\left(\frac{2}{3},\frac{1}{3}\right)}{9^{\frac{1}{3}}}$.